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S-2632

M. Sc. (Sem. I) Examination

March / April – 2011

Mathematics : 401

(Measure Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. (Sem. 1)**

Name of the Subject :
☛ **Mathematics : 401**

☛ Subject Code No. : **2 6 3 2** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Attempt all questions.
(3) Each question is of 14 Marks.
(4) Marks are equally distributed.
(5) figures to the right indicates full Marks.

1 Answer any **two** from the following. **14**

(a) Let F be a closed and bounded set of real numbers.Then prove that each open covering of F.has a finite subcovering.

(b) Define outer measure of a set and prove that the outer measure of a set is tanslation invariant.

(c) If E_1 and E_2 are measurable then prove that,

(i) $E_1 \cup E_2$ is also measurable and

(ii) $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

2 Answer any **two** from the following : **14**

(a) Define an open set and a closed set. Prove that the complement of an open set is closed and the complement of a closed set is open.

(b) Let $\langle F_n \rangle$ be a sequence of measurable functions with the same domain of definition then prove that the functions $\sup\{f_1, f_2, \ldots f_n\}$, $\inf\ \{f_1, f_2, \ldots f_n\}$, $\sup_n f_n$, $\inf_n f_n$, $\overline{\lim} f_n$ and $\underline{\lim}\, f_n$ are all measurable.

(c) Prove the following :

(i) Outer measure of a singleton set is zero.

(ii) If A is countable then $m^*A = 0$.

(iii) $m^*(A \cup B)m^*B \; if \; m^*A = 0.$

3 Answer any **two** from the following. 14

(a) Let A be any set ,and $E_1, E_2, E_3.....E_n$ a finite sequence of disjoint measurable sets. Then prove that

$$m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_1\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_1).$$

(b) If f is integrable on $[a,b]$ and $F(x) = F(a) + \int_a^x f(t)dt$ then prove that $F'(x) = f(x)$.

(c) Prove that the interval (a, ∞) is measurable.

4 Answer any **two** from the following. 14

(a) If f and g are integrable over E,then prove that:

(i) The function cf is integrable over $E, and \int_E cf = c\int_E f.$

(ii) The function $f + g$ is integrable over E and $\int_E (af + bg) = a\int_E f + b\int_E g..$

(iii) If $f \leq g$ a. e., then $\int_E f \leq \int_E g.$

(b) If $A \in \mathcal{A}$,where $\mathcal{A}$ is an algebra of sets then prove that A is measurable with respect to μ^*.

(c) Let g be integrable over E and suppose that $\langle f_n \rangle$ is a squence of measurable functions such that on E; $|f_n(x)| \leq g(x)$ and or almost all x in $E \; f_n(x) \to f(x)$. Then prove that $\int_E f = \lim \int_E f_n$.

5 Answer any **two** from the following. 14

(a) If $E_1 \in \mathcal{B}$, $\mu E_1 < \infty$ and $E_1 \supset E_{i+1}$,then prove that

$$\mu\left(\bigcap_{i=1}^{n} E_1\right) = \lim_{n \to \infty} \mu E_n.$$

(b) Let f be a nonnegative function which is integrable over a set E. Then prove that for given $\varepsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta; \int_A f < \varepsilon$.

(c) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathcal{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$ .Then prove that there is a unique measurable extended real valued function f on x such that $f \leq a$ on B_α and $f \geq \alpha$ on $X - B_\alpha$.
