SB-3449/300

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M. A. / M. Sc. (Part-I) (Mathematics) Examination

March / April – 2011

Paper-401 : Real Analysis

(Old Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M.A./M. Sc. (Part-I)**

Name of the Subject :
☛ **401 -Real Analysis**

☛ Subject Code No. : **3 4 4 9** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.
(3) Follow usual conventions and notations.
(4) Figure to the right indicates marks.

1 (a) Prove that the interval (a,∞) is measurable. **6**

(b) Define outer measure, Prove that outer measure is translation invariant. **4**

(c) Prove that $m^{*}(A)=0$ then $m^{*}(A\cup B)=m^{*}(B)$. **4**

OR

1 (a) Define length of an interval. Prove that outer measure of an interval is its length. **6**

(b) If $\{A_1, A_2, A_3, ...\}$ is a countable family of subset of R, then prove that $m^{*}(U_{n=1}^{\infty}A_n) \le \sum_{n=1}^{\infty} m^{*}(A_n)$. **4**

(c) Prove that outher measure of the set of all rational numbers is zero. **4**

2 (a) Define σ-algebra. Prove that the collection of all measurable sets is a σ-algebra. **6**

(b) Define constant function. Prove that the constant function define over a measurable function is measurable. **4**

(c) Let $\langle f_n \rangle$ be an increasing sequence of non-negative measurable functions and let $\langle f_n \rangle$ converges to f almost everywhere. If then prove that $\int_E f = \lim \int_E f_n$. **4**

OR

2 (a) If p and q are nonnegative extended real numbers such that $\frac{1}{p}+\frac{1}{q}=1$ and $f \in L^p$ and $g \in L^q$ then prove that $fg \in L^1$ and $\int |fg| \le \| f \|_p \| g \|_q$. **6**

(b) Let f be a non-negative function, integrable over a set E. Then prove that, given $\in > 0$ there is a $\delta > 0$, such that for every set $A \subset E$ with $m(A) < \delta$, we have $\int_A f < \in$. **4**

(c) Let $f \in L^p[a,b], g \in L^p[a,b]$ then prove that $f+g \in L^p[a,b]$. **4**

3 (a) Define simple function. Let f be defined and bounded on a measurable set E with $m(E) < \infty$. If f is measurable then prove that

$\inf_{f \le \psi} \int \psi(x)dx = \inf_{\phi \le f} \int \phi(x)dx$ for all simple functions ψ and ϕ. **6**

(b) State and prove monotone convergence theorem for non-negative measurable functions. **4**

(c) If $f \in L^1$ and $g \in L^\infty$ then prove that $\int |fg| \le \|f\|_1 \|g\|_\infty$. **4**

OR

3 (a) Define Lebesgue integral of bounded function. If f and g are bounded measurable functions on set E of finite measure and $f \le g$ almost everywhere then prove that $\int_E f \le \int_E g$. Henceprove that $\left|\int_E f\right| = \int_E |f|$. 6

(b) Prove that sum and difference of two absolutely continuous function is also absolutely continuous. 4

(c) If f is measurable function and f=g almost everywhere then prove that g is also measurable function. 4

4 (a) Prove that a function f is of bounded variation on $[a,b]$ if and only if f is difference of two Monotonically increasing real valued function defined on $[a,b]$. 6

(b) If $c \in (a,b)$, if $f \in Bv([a,c])$ and $f \in Bv([c,b])$ then prove that $f \in Bv([a,b])$ and $T_a^b(f) = T_a^c(f) + T_c^b(f)$. 4

(c) Define convex function. Prove that convex function is absolutely continuous function. 4

OR

4 (a) If f is bounded and measurable on $[a,b]$ and $F(x) = \int_a^x f(t)dt$ then prove that $F'(x) = f(x)$ for almost all x in $[a,b]$. 6

(b) Prove that if $A \le f(x) \le B$ then, $Am(E) \le \int_E f \le B\,m(E)$ for bounded measurable functions define on a set E of finite measure. Where A and B are constants. 4

(c) Define measure space (X, β). If $E_i \in \beta, \mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu E_n$. 4

5 (a) Let v be a signed measure on a measurable space (X,β). Then prove that there is a positive set A and a negative set B such that $X = A \cup B$ and $A \cap B = \phi$. 6

(b) Let f be a non-negative measurable function then show that $\int_E f = 0$ implise that $f = 0$ almost everywhere. 4

(c) Prove that the union of a countable collection of a positive set with respect to signed measure is again positive with respect to signed measure. 4

OR

5 (a) Prove that for an arbitrary Hahn decomposition {A, B} of (x,β,ϑ), if we define two set functions $v^+(E)=\vartheta(E\cap A)$ and $v^-(E)=-\vartheta(E\cap B)$ for $E\in\beta$, then $\{v^+,v^-\}$ is Jordan decomposition for (X,β,ϑ). **6**

(b) If A and B are disjoint measurable subset of [a,b] and if f is bounded Lebesgue-integrable function on [a,b] then $\int_{A\cup B} f = \int_A f + \int_B f$. 4

(c) Define measurable rectangle. Let $\{(A_i x B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle A×B then $\lambda(A\times B)=\sum\lambda(A_i\times B_i)$. 4