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SB-2737

M. Sc. (Sem - II) Examination

March / April – 2011

Mathematics : Paper - 506

(Functions of Complex Variables)

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. (Sem - II)**

Name of the Subject :
☛ **Mathematics : Paper - 506**

☛ Subject Code No. : **2 7 3 7** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) This paper contains five questions.

(3) All questions are compulsory.

(4) Notations used are standard.

1 Answer any **two** form the following : **14**

(a) If $|z| \leq ½$ then prove that, $|\log E_n(z)| \leq 2|z|^n$.

(b) Show that (in usual notations) $\frac{\Pi^2}{Sin^2 \Pi z} = \sum_{n \in Z} \frac{1}{(z-n)^2}$.

(c) Given a fractional linear map F, prove that there exist complex numbers α, β, γ such that either $F = \alpha z + \beta$ or $F(z) = T_\gamma 0 M_\alpha 0 J 0 T_\beta$.

2 Answer any **two** from the following : **14**

(a) Let f be an entire function of strict order $\leq \rho$, , Let $V_f(R)$ be the number of zeros of f in the disc of radius R. Then prove that $V_f(R) << R^\rho$.

(b) Let f be an entire function of order ρ, and $let\langle Z_n \rangle$ be the sequence of its $z\cos \neq 0$. Let k be the smallest integer > ρ. Let $P = P_k$. Then prove that

$f(z) = e^{h(z)} z^m \Pi\left(1 - \frac{z}{z_n}\right) e^{p\left(\frac{z}{z_n}\right)}$, where m is the order of f at 0, and h is a palynomial of degree $\leq \rho$.

(c) Prove that $\sum_{n=1}^{\alpha} \frac{1}{n^2} = \frac{\pi^2}{\sigma}$.

3 Answer any **two** from the following : **14**

(a) Show that,

(i) An elliptic function without poles is constant.

(ii) For given $z \in C$, there exists a unique element $z_0 \in P$. Such that $Z \equiv z_0$ mod L.

(b) Let P be a fundamental parallelogram and assume that the elliptic function f has no zero or no pole on its boundary. Let $\{ai\}$ be the singular points of f inside p and let f have order mi at ai. Then prove that $\sum miai \equiv (mod\ L)$.

(c) Show that, $\rho(z) = \frac{1}{z^2} + \sum_{n=1}^{\infty} (zn+1)\ S_{2n+2}(L)\ Z^{2n}$.

4 Answer any **two** from the following : **14**

(a) In usual notations prove that the Euler constant γ is given by $\gamma = \lim_{n \to \infty} \left[\sum_{k=1}^{n} \frac{1}{k} - log\, n \right]$

(b) show that, (i) $Res_{-n}\Gamma(z) = \frac{(-1)^n}{n!}$

(ii) $\Gamma(z+1) = z\Gamma(z)$

(c) In usual notations prove that, $\Gamma^1 / \Gamma(z) = \int_0^{\infty} \left(\frac{e^{-t}}{t} - \frac{e^{-zt}}{1-e^{-t}} \right) dt$ for $R_e(z) > 0$.

5 Answer any **two** from the following : **14**

(a) Let f be holomarphic on a closed disc of radius R, centered at the origin. Let $\| f \|_r = \max | f(z) |$ for $| z | = r < R$.

Then Prove that, $\| f \|_r \leq \frac{2r}{R-r} \, sup_R \operatorname{Re} f + \frac{R+r}{R-r} | f(0) |$.

(b) Let $f : D \to D$ be an analytic function of the unit disc into it self such that $f(0) = 0$. Then prove that,

(i) $| f(z) | \leq | z |$ for all $Z \in D$.

(ii) If for some $Z_0 \neq 0, | f(z_o) | = | z_o |$ then there is some complex number α of absolute value 1 such that, $f(z) = \alpha z$.

(c) Let f be an entire function. Then for $r < R$. prove that,

$M_f(r) \leq \frac{R+r}{R-r} m_f(R)$ and in particular, $M_f(r) \leq 3 m_f(zr)$