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SB-2732

M. Sc. (Sem. - II) Examination

March / April – 2011

Paper - 501 : Mathematics
(Differential Geometry)

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. (Sem. II)**

Name of the Subject :
☛ **Mathematics : Paper-501**

☛ Subject Code No. : **2 7 3 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Follow usual notations.

(4) Marks are equally distributed.

1 (a) Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature bears a constant ratio with its torsion at any point. 7

(b) Find curvature and torsion for the circular helix $x = a\cos(\theta); y = a\sin(\theta); z = a\theta\cot(\alpha)$ **7**

OR

(b) For the space curve in usual notations prove that : **7**

$[t't''t'''] = k^3[kT' - k'T]$.

2 (a) State equation of involute and derive its formula to find curvature and Torsion . **7**

(b) Find the envelop of family of plances 7

$3a^2x - 3ay + z = a^3$. Also show that its edge of regression is the curve intersecting surfaces $xz = y^2$ and $xy = z$.

OR

(b) Find the equation of involute and evolute of the circular helix. $\bar{r} = (a\cos\theta; a\sin\theta; a\theta\tan\alpha)$ **7**

3 (a) State and prove Serret-Frenet formula. **7**

OR

(a) Define osculating plane. Derive equation of osculating plane. Find the osculating plane at the point (0,0,0) on the helix : $x = a\cos\theta; y = a\sin\theta; z = a\theta$ **7**

(b) In usual notations prove that : **7**

$$x'''^2 + y'''^2 + z'''^2 = \frac{1}{(\sigma\rho)^2} + \frac{\rho'^2 + 1}{\rho^4}.$$

4 (a) Define first and second curvature. Derive equation to determine principal curvature in the form : **7**

$H^2k^2 - (EN - 2FM + GL)k + T^2 = 0$.

(b) If ψ is an angle between two directions given by $Pdu^2 + 2Qdud\vartheta + Rd\vartheta^2 = 0$ then prove that **7**

$$\tan(\psi) = \frac{2H\sqrt{Q^2 - PR}}{ER - 2FQ + GP}$$

OR

(b) If two parametric curves through any point of surface cut an angle w then prove that $\tan(w) = \frac{H}{F}$. **7**

5 (a) State and prove Euler's theorem on normal curvature. **7**

OR

(a) Define conjugate direction. Obtain an analytical expression for two directions (du, dv) and $(\delta u, \delta v)$ to be conjugate. **7**

(b) Define the terms : **7**

(i) Space curve.

(ii) Curvature of a space curve

(iii) Torsion of a space curve.

State and prove necessary and sufficient conditions for a space curve to be a (i) Straight line, (ii) Plane cruve.
